CHAPTER Ⅳ

COEFFICIENTS FOR INPUT-OUTPUT ANALYSIS
AND COMPUTATION METHODS

§ 1 Input Coefficients

1 Calculating Input Coefficients

"Input coefficients" represent the scale of raw materials and fuels used can be obtained by dividing the input of raw materials and fuels utilized to generate one unit of production in each sector. They correspond to basic unit prices, and are obtained by dividing the amount of raw materials, fuel, etc. input into each sector by the domestic production value of that sector. A list of input coefficients indicated for each industry is referred to as an "input coefficient table."

(Note) The Input-Output Tables are basically "commodity-by-commodity" tables. The "sectors" comprising the endogenous sectors at the top and side of the table represent types of goods and services produced by the industries, producers of government services, and producers of private non-profit services for households. For the sake of convenience, they are referred to as "industries" or "industrial sectors."

To simplify, if the national economy is deemed to be comprised only of Industry 1 and Industry 2, the Basic Transaction Table may be as indicated in Chart 4-1.

Chart 4-1 Basic Transaction Table (Model 1)

	Industry 1	Industry 2	Final demand	Total domestic products
Industry 1	x_{11}	x_{12}	F_1	X_1
Industry 2	x_{21}	x_{22}	F_2	X_2
Gross value added	V_1	V_2		
Total domestic products	X_1	X_2		

Where

Supply-demand balance equation (balancing of total supply and total demand)

$$\begin{cases} x_{11} + x_{12} + F_1 = X_1 \\ x_{21} + x_{22} + F_2 = X_2 \end{cases}$$

Income-expense balance equation

$$\begin{cases} x_{11} + x_{21} + V_1 = X_1 \\ x_{12} + x_{22} + V_2 = X_2 \end{cases}$$

When "a_{11}" is defined as the figure produced by dividing "X_{11}," representing the input of Industry 1 from Industry 1 by "X_1," representing the domestic production, " a_{11}" represents the input required to produce one unit of production of Industry 1 from Industry 1.

$$a_{11} = \frac{x_{11}}{X_1} \quad .... [1]$$

Similarly, the expression " $a_{21} = \frac{x_{21}}{X_1}$ " represents the amount of raw materials, etc. that the Industry 1 input from Industry 2 to produce one unit of the product.

Similar to intermediate inputs, " $v_1 = \frac{V_1}{X_1}$ " can be defined by dividing the value added produced in Industry 1 by domestic production.

In this case, "V_1," the gross value added, signifies inputs of the primary factors of Sector 1, such as labor and capital, and "v_1" can be regarded as an input unit of such production factors.

Applying the above procedure to Industry 2 (the second column for Chart 4-1) produces the following input coefficient table (Chart 4-2)

Chart 4-2　Input Coefficient Table (Model)

	Industry 1	Industry 2	Note
Industry 1	a_{11}	a_{12}	$a_{ij} = \dfrac{x_{ij}}{X_j}$
Industry 2	a_{21}	a_{22}	
Gross value added	v_1	v_2	$v_j = \dfrac{V_j}{X_j}$
Total domestic products	1.0	1.0	

　　　Indicating the scale of raw materials, etc. required to generate one unit of production in each sector, the input coefficient table can be referred to as the basic production unit table. The sum of input coefficients including the gross value added portion in each sector is defined as 1.0.This series of calculations is made for Basic Transaction Tables for 13 sectors in the 2005 Input-Output Tables, and indicated in Table 1-(2) in Document 2 of Chapter 10.

　　　For instance, looking at the top of the table along the agricultural, forestry, and fisheries, when the agricultural, forestry, and fisheries industry generates one unit of production, intermediate inputs of 0.124901 units were produced by the agricultural, forestry, and fisheries sector, and, 0.194886 units of intermediate inputs were similarly produced by the manufacturing sector. Thus, a total of 0.471563 units of intermediate inputs were required. The table also indicates that 0.528437 units of gross value added were produced as the result of the production.

(Note)　　Ideally, "Unit" here should be a physical unit, such as a weight or number of items, etc. In the Input-Output Tables, figures are represented in monetary amount to maintain consistency for various products. The input coefficients calculated from these figures are the input coefficients based on monetary values at the prices of the relevant year.

　　　Suppose production of 100-yen of Product A requires 50 yen of Product B. If the prices of all products can be expressed through "amount-by-unit price," this situation may be equivalent to a hypothetical situation in which 50 of "Product B that can be purchased at one yen" was input to produce 100 of "Product A that can be purchased at one yen." Production volumes of all industries are valued at the unit of quantity equivalent to one yen (or one dollar or one million yen or other consistent monetary units), to allow comparison of industry production units. This system is called Input-Output Tables at the "yen value unit." Valuation by the "yen value unit" for the base year represents the nominal value itself. If the "yen value unit" in the base year is applied to the year to be compared, "real evaluation" based on the valuation at yen value in the table for the base timetable can be obtained.

2　Definition of Input Coefficients

(1)　Measurement of Effects of Input Coefficients on Production

　　　Next, the meanings of input coefficients are considered with Chart 4-1 and Chart 4-2 mentioned above.

　　　Suppose demand for Industry 1 has increased by one unit. Industry 1 will require raw materials, etc. to generate one unit of production. Industry 1 will thus generate intermediate demands of "a_{11}" and "a_{21}" units of raw materials to Industry 1 and Industry 2, respectively, in accordance with the input coefficients, which is the primary production repercussion. Receiving the demands, Industry 1 and Industry 2 will further generate the secondary production repercussions, in accordance with the respective input coefficients to produce "a_{11}" and "a_{12}" units. This series of production repercussions continues infinitely, until domestic production levels for the respective sectors can ultimately be calculated as the summation of all production repercussions.

　　　In this manner, input coefficients are crucial to measuring how much production can be ultimately induced at each sector when certain levels of final demand are generated in an industrial sector.

　　　However, it is all but impossible and unfeasible to trace and calculate each process of production repercussion occurrences. The following inverse matrix coefficients are prepared to simplify such production repercussion calculations. As a preparatory step, it is necessary to explain the process of production repercussions.

(2)　Mathematical Computation of Effects on Production

　　　In Chart 4-1 above, the mathematical formula of the balance for every row is described by the following equations:

$$\left. \begin{array}{l} x_{11} + x_{12} + F_1 = X_1 \\ x_{21} + x_{22} + F_2 = X_2 \end{array} \right\} \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots [2]$$

　　　As in the case of equation [1], "a_{21}," "a_{12}," and "a_{22}" are calculated and substituted into equation [2], resulting in the following modifications:

$$a_{11}X_1 + a_{12}X_2 + F_1 = X_1$$
$$a_{21}X_1 + a_{22}X_2 + F_2 = X_2$$ $\left.\begin{array}{c}\\\\\end{array}\right\}$ ································ [3]

As indicated in equation [3], certain relationship exists between final demand and domestic production. The relationship is defined by "input coefficients."

Equation [3] can be expressed in a matrix, as follows:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

This is referred to as the input coefficient matrix.

Assigning specific figures to the final demands represented by "F_1" and "F_2" in the simultaneous equations of [3] and solving them makes it possible to obtain domestic production that meets final demand. This calculation produces the domestic production levels in Industry 1 and Industry 2 resulting from production repercussion effects.

Demand increases in a certain industrial sector will require inputs of raw materials and fuels, etc. from other industries for production activities, and thereby affecting not just industry production but those of the other industries, which will further generate additional demands in the original sector as repercussion effects. Equation [3] indicates a mechanism for calculating the cumulative effects of these repercussions. This is the fundamental approach and constitutes the basis of input-output analyses based on input coefficients.

However, note that this approach is based on the premise of stable input coefficients, as indicated below. Constant fluctuations in input coefficients will make it impossible to determine consistent relationships between final demand and domestic production.

3   Stability of Input Coefficients

(1)   Consistency of Production Technology Levels

In the Input-Output Tables, input ratios of raw materials and fuels, etc. required to produce goods and services represented by the input coefficients are assumed not to fluctuate significantly between the year to be analyzed and that in which the table is compiled.

Input coefficients, in short, reflect production technologies adopted in a certain year. Changes in production technologies may naturally change the input coefficients.

Although drastic changes are generally not supposed to occur in production technologies in short timeframes, in countries such as Japan, extremely rapid technological advancements may make it necessary to acquire information on changes in input coefficients and make proper adjustments by some method.

(2)   Consistency of Production Scale

Each industrial sector is comprised of various enterprises and establishments with different production scales. Even if the same products are produced, different production scales will inevitably lead to different input coefficients due to the different technologies and economy-of-scale levels.

However, the Input-Output Tables are compiled while reflecting the economic structures in the compilation years. In input-output analyses, production scales of enterprises and establishments, allocated to respective industrial sectors, are assumed not to undergo significant changes between the years to be analyzed and those in which tables are compiled.

(3)   Change Factors of Input Coefficients

It is assumed that there are few changes in input coefficients between the year to be analyzed and the compilation year. However, in addition to the (1) and (2) above, the following factors may change over time:

[1]   Changing Relative Prices

Since individual transactions in the Basic Transaction Tables are valued at prices in the year when the tables are compiled, changing the relative prices of goods and services will change the input coefficients, even if the technological structures remain constant.

Historical comparisons would require Linked Input-Output Tables based on fixed price valuations, in which effects of fluctuating relative prices are eliminated.

[2]   Changing Product Mixes

If products with different input structures and unit prices are placed in the same sector (which is referred to as a "product mix"), changes in product structures within the sector will change the input coefficients of the entire sector, even if there is no change in input structure or unit price of each product.

§2   Inverse Matrix Coefficients

1   Definition and Computation of Inverse Matrix Coefficients

One of the important analyses in input-output analyses is to analyze the direct and indirect effects of certain final demands that occurred in an industrial sector on other industrial sectors. As stated before, input coefficients in the respective industrial sector may play crucial roles.

Suppose the national economy is comprised only of Industry 1 and Industry 2. As stated in section 1, when the final demand is given, solving the following simultaneous equations will give the domestic production levels of Industry 1 and Industry 2.

$$\left.\begin{array}{l} a_{11}X_1 + a_{12}X_2 + F_1 = X_1 \\ a_{21}X_1 + a_{22}X_2 + F_2 = X_2 \end{array}\right\} \dots\dots\dots\dots\dots\dots\dots\dots [3]$$

Indeed, if the entire structure were composed only of these two sectors, calculations would be quite simple. In reality, even the Medium Consolidated Sector Classification has as many as 108 sectors, which makes solving simultaneous equations for all of them impractical and makes it almost impossible to conduct proper analyses.

If calculations can be made in advance, as to what kind of production repercussions on various sectors may be expected if one unit of final demand is produced for a certain sector, and how much domestic production will be finally expected in each sector, analyses could be significantly expedited. "Inverse matrix coefficient tables" are compiled in response to this need.

In the matrix indication for equation [3] above,

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} \dots\dots\dots\dots\dots (3)'$$

when the input coefficient matrix is defined as

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = A$$

the final demand column vector is defined as

$$\begin{bmatrix} F_1 \\ F_2 \end{bmatrix} = F$$

and the domestic production column vector is defined as

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = X,$$

$$AX + F = X \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots [3]''$$

can be obtained. The solution for X is

$$X - AX = X$$
$$(I - A)X = F$$
$$\therefore X = (I - A)^{-1}F$$

where "I" is a Identity matrix, $(I - A)^{-1}$ is the inverse matrix of $(I - A)$, as follows:

$$(I - A)^{-1} = \begin{bmatrix} 1 - a_{11} & -a_{12} \\ -a_{21} & 1 - a_{22} \end{bmatrix}^{-1}$$

The factors of this matrix are referred to as "inverse matrix coefficients," a listing of which is the "Inverse Matrix Coefficient Table." This table indicates how much production will be ultimately induced in what industry by a demand increase of one unit in a certain industry. Once the inverse matrix coefficients are calculated, the simultaneous equations in

[3] do not need to be solved independently. When the final demand in a sector is given, the domestic production at each sector, corresponding to the final demand, can be immediately calculated.

(Note) For the equation of [3], to be able to give a non-negative solution for a certain F (non-negative), the necessary and sufficient condition will be that all principal minors in the matrix $(I - A)$ in the matrices need to be positive (Hawkins-Simon's condition). For all the principal minors in matrix $(I - A)$ to be positive, the sufficient condition will be

$$\sum_{i=1}^{n} a_{ij} \quad < \quad 1 \quad (j = 1,2,\cdots\cdots, \quad n)$$

Here the sum of input coefficients should always be less than 1 (Solow's condition). That is necessary condition.

For the 13-sector Basic Transaction Tables for the 2005 Input-Output Tables, calculations for the inverse matrices of the type of $[I - (I - \hat{M})A]^{-1}$ (please refer to the following explanations) are indicated in Table 1-(3) in Document 2 of Chapter 10.

Sectors at the top of the inverse matrix coefficient table are those in which one unit of the final demand has been generated; sectors at the side indicate those in which production can somehow be induced by generation. For instance, in examining the agriculture, forestry, and fisheries from the top of the table down, one unit of final demand in the agriculture, forestry, and fisheries industry can ultimately generate 1.127988 units of production inducement in the agriculture, forestry, and fisheries industry itself; and production inducements in the mining, manufacturing, and construction industries will be 0.001004 units, 00.336313 units, and 0.010306 units, respectively, resulting in a total of 1.809162 units of production inducements, which can be interpreted as corresponding to the vertical sum.

Input coefficients introduced in §1 indicate the amount of raw materials and other factors directly required to produce one unit of certain goods or services. The inverse matrix coefficients indicate the magnitude of the ultimate direct and indirect production repercussions on various industrial sectors when there is one unit of final demand for a certain sector.

(Note) In this way, when inverse matrix coefficients are observed in relation to production repercussions, for instance, when one unit of final demand is generated in agriculture, forestry, and fisheries, production in the industry must increase (direct effect) to satisfy demand.

Due to agriculture, forestry, and fisheries to increase production, other sectors must increase production, the effects of which further increase production in agriculture, forestry, and fisheries (indirect effects). As a result, the production increase in the agriculture, forestry, and fisheries industry usually exceeds one unit. The diagonal elements in the inverse matrix coefficients indicating the production increase in the self-activity sector commonly exceed 1.

A column vector with the inverse matrix defined as B, the diagonal element as b_{ii}, and the column vector as (u_i), in which the i-th element is 1 and the other elements are 0, can be describe as follows:

$$Bu_i = \begin{bmatrix} b_{11} & \cdots & b_{1i} & \cdots & b_{1n} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ b_{i1} & \cdots & b_{ii} & \cdots & b_{in} \\ \vdots & \ddots & & \ddots & \vdots \\ b_{n1} & \cdots & b_{ni} & \cdots & b_{nn} \end{bmatrix} \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} b_{1i} \\ \vdots \\ b_{ii} \\ \vdots \\ b_{ni} \end{bmatrix}$$

It can be concluded from the above that the i-th column vector of the inverse matrix B indicates the production increase units at each sector when one unit of final demand is generated in the sectors. (For the reasons mentioned, $b_{ii} \geq 1$)

The vertical sum of aggregated i-th column in the inverse matrix B corresponds to the production inducement coefficient of the i-th sector (please refer to §3).

2　Types of Inverse Matrix Coefficients (handling of imports)

In analyses of production repercussions with Input-Output Tables, a major issue is import handling. §1 above the mentioned the so-called Type $(I - A)^{-1}$ model, which is a simplified model excluding imports. Basically various goods are imported and consumed in parallel with domestic products in industries and households.

Chart 4-3 shows the model for Basic Transaction Tables, clearly indicating imports. For row items, both intermediate demand (X_{ij}) and final demand (F_i) are supplies including imports, and columns and rows (production) offset each other because imports are indicated negative values.

Chart 4-3　Basic Transaction Table (Model 2)

	Industry 1	Industry 2	Final demand	Import	Domestic production
Industry 1	x_{11}	x_{12}	F_1	$-M_1$	X_1
Industry 2	x_{21}	x_{22}	F_2	$-M_2$	X_2
Gross value added	V_1	V_2			
Domestic production	X_1	X_2			

Input coefficients include imports. This implies that all repercussions derived from final demand do not necessarily induce domestic production; some effects may induce imports.

In other words, for accurate determination of domestic production inducements, import inducements must be deducted.

It is thus necessary to provide a calculation method for inverse matrix coefficients that accounts for import inputs.

The inverse matrix coefficients in the "$[I - (I - \hat{M})A]^{-1}$ Type" are commonly utilized in Japan. Several inverse matrix coefficient calculation methods are also used, as follows:

(1)　$(I - A)^{-1}$ Type

This type is presented as a simplified model excluding imports in "1" above. In this model, imports are handled exogenously.

In the basic model (two rows and two columns), the supply-demand balance equation can be presented as follows:

$$\left. \begin{array}{l} a_{11}X_1 + a_{12}X_2 + F_1 - M_1 = X_1 \\ a_{21}X_1 + a_{22}X_2 + F_2 - M_2 = X_2 \end{array} \right\} \cdots\cdots\cdots [4]$$

The matrix denotation is as indicated below.

$$AX + F - M = X \dots\dots\dots\dots\dots\dots\dots\dots [4]'$$

This is a "Competitive import type" model in which intermediate demand (AX) and final demand (F) include a certain volume of imports.

The solution for X is:

$$X - AX = F - M$$
$$(I - A)X = F - M$$
$$\therefore X = (I - A)^{-1}(F - M)$$

In this model, both final demand and imports can be determined exogenously. Imports, however, can be induced by domestic production, except in certain special circumstances. In other words, it is to regard them as endogenously determined. Thus, this model is used infrequently.

(2)　$[I - (I - \hat{M})A]^{-1}$ Type

This model divides final demand (F) into domestic final demand (Y) and export (E), giving the following equation:

$$F = Y + E$$

This is substituted into [4]' above. The supply-demand balance equation can be expressed as follows:

$$AX + Y + E = X \dots\dots\dots\dots\dots\dots\dots\dots [5]$$

In the tables, mere transit transactions are not supposed to be incorporated into exports. Thus, it can be assumed that exports do not include imports. Import coefficients by row can be defined as follows:

$$m_i = \frac{M_i}{\sum_j a_{ij}X_j + Y_i}$$

In other words, "m_i" represents the ratio of imports in product "i" within total domestic demands, or ratios of dependence on imports; while ($1 - m_i$) represents self-sufficiency ratios.

When [5] is represented for "i" row,

$$\sum_j a_{ij}X_j + Y_i + E_i - M_i = X_i \dots\dots\dots\dots\dots [6]$$

From the definition of import coefficients,

$$M_i = m_i \left(\sum_j a_{ij} X_j + Y_i \right) \quad \text{................................ [7]}$$

[7] is substituted into [6], and the equation is as follows:

$$X_i - (1 - m_i) \sum_j a_{ij} X_j = (1 - m_i) Y_i + E_i \quad \text{........ [8]}$$

The diagonal matrix ($\hat{M}$) can be assumed to have an import coefficient (m_i) as the diagonal element and zero as the non-diagonal element.

$$\hat{M} = \begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{bmatrix}$$

From [8] above, the following equation can be obtained:

$$[I - (I - \hat{M})A] \ X = (I - \hat{M})Y + E \quad \text{.................. [9]}$$

From [9], the following equation can be obtained:

$$X = [I - (I - \hat{M})A]^{-1}[(I - \hat{M})Y + E] \quad \text{..............[10]}$$

Giving domestic final demand (Y) and export (E) produces domestic production (X).

Here, $(I - \hat{M})A$ indicates the input ratio of domestic products when the import input ratio is assumed to be constant in all sectors, whether they are for intermediate demand or final demand. $(I - \hat{M})Y$ indicates domestic final demand for domestic products under the same assumption. In other words, this is the "competitive import type" model when import ratios for individual items (for rows) (or import coefficients) are assumed to be identical in all output sectors.

Inverse matrix coefficient tables based on this model are commonly used in Japan. Table 1-(3) in Document 2 of Chapter 10 compiles the 13-sector Basic Transaction Tables for the 2005 Input-Output Tables, based on this approach.

(3) $(I - A^d)^{-1}$ Type

The inverse matrix coefficients based on this model is the "non-competitive import type," which can be used to analyse when the input ratios of imports differ from sector to sector.

Chart 4-4 shows simplified non-competitive Import Basic Transaction Table.

Chart 4-4　Basic Transaction Table (Model 3)

		Industry 1	Industry 2	Final demand	Import	Domestic production
Domestic	Industry 1	x_{11}^d	x_{12}^d	F_1^d	—	X_1
	Industry 2	x_{21}^d	x_{22}^d	F_2^d	—	X_2
Import	Industry 1	x_{11}^m	x_{12}^m	F_1^m	$-M_1$	—
	Industry 2	x_{21}^m	x_{22}^m	F_2^m	$-M_2$	—
Gross value added		V_1	V_2			
Domestic production		X_1	X_2			

Naturally, the following equations can be defined:

$$x_{ij} = x_{ij}^d + x_{ij}^m$$
$$F_i = F_i^d + F_i^m$$

The supply-demand balance for domestic products can be presented as follows:

$$\left. \begin{aligned} x_{11}^d + x_{12}^d + F_1^d &= X_1 \\ x_{21}^d + x_{22}^d + F_2^d &= X_2 \end{aligned} \right\} \quad \text{............................. [11]}$$

Where input coefficient for domestic intermediate goods is defined as follows:

$$a_{ij}^d = \frac{x_{ij}^d}{X_j}$$

Then, the equations [11] can be as follows:

$$\left.\begin{array}{l} a_{11}^d X_1 + a_{12}^d X_2 + F_1^d = X_1 \\ a_{21}^d X_1 + a_{22}^d X_2 + F_2^d = X_2 \end{array}\right\} \quad \text{...................[11]'}$$

This can be represented by the following matrix:

$$A^d X + F^d = X \text{...[11]''}$$

This is the "non-competitive import type" model. Both intermediate demand ($A^d X$) and final demand (F^d) cover domestic products and exclude imports.

The solution of [11]'' for X is as follows:

$$X - A^d X = F^d$$
$$(I - A^d)X = F$$
$$\therefore X = (I - A^d)^{-1} F^d$$

When the final demand for domestic products (F^d) is given, the domestic production level (X) can be obtained:

The relationship with the competitive import type model may be presented as follows: When the input coefficient matrix for import is defined as (A^m) and the final demand colum vector for imports is defined as (F^m), the following equations can be derived:

$$A = A^d + A^m$$
$$F = F^d + F^m$$

Based on the above equations, the following supply-demand balance can be obtained:

$$(A^d + A^m)X + (F^d + F^m) = X + M$$

This is the basic equation of the competitive import type of model.

In the actual economy, input ratios of domestic and imported products may generally differ from sector to sector. Inverse matrix coefficients based on this model represent this situation as is. When this type of inverse matrix coefficients are compared with (2) $[I - (I - \hat{M})A]^{-1}$, significant differences may be observed at times in certain sectors.

In the Input-Output Tables compiled as a five-year project by ten authorities, inputs and outputs are divided into domestic and imported products, making it possible to use two different types of inverse matrix tables. The appropriate one will depend on the purposes of analyses and considerations regarding consistency with assumptions.

3   Index of the Power of Dispersion and Index of the Sensitivity of Dispersion

(1)   Index of the Power of Dispersion

The figure in each column in the inverse matrix coefficient table indicates the production required directly and indirectly at each row sector when the final demand for the column sector (that is, demand for domestic production) increases by one unit. The total (sum of column) indicates the scale of production repercussions on entire industries, caused by one unit of final demand for the column sector.

The vertical sum of every column sector of the inverse matrix coefficients is divided by the mean value of the entire sum of column to produce a ratio. This ratio indicates the relative magnitudes of production repercussions; that is, which sector's final demand can exert the greatest production repercussions on entire industries. This is called the "Index of the Power of Dispersion" and can be calculated as follows:

$$\text{Index of the power of dispersion by sector} = \frac{\text{Each sum of cloumn in inverse matrix coefficient table}}{\text{Mean value of entire vertical sum in the inverse matrix coefficient table}}$$

$$= \frac{b_{*j}}{\overline{\overline{B}}}$$

Here,

$$b_{*j} = \sum_i^n b_{ij}$$

$$\overline{\overline{B}} = \frac{1}{n}\sum_j b_{*j} = \frac{1}{n}\sum_i \sum_j b_{ij}$$

(Please refer to Chart 4-5)

The index of the power of dispersion indicated above is referred to as the "first category index of the power of dispersion." Table 4-1 indicates the calculation of the index of the power of dispersion by utilizing $[I-(I-\hat{M})A]^{-1}$ as the inverse matrix in the 34-sector table of the 2005 Input-Output Tables. This indicates that the indices for iron and steel and transportation machinery, etc. have relatively high indices of the power of dispersion, indicating that both sectors exert great production repercussions on entire industries.

Conversely, sectors indicating low indices of the power of dispersion are real estate, petroleum and coal products, education and research and so forth. Service-related sectors generally have slight production repercussions on entire industries.

However, the sum of column of inverse matrix coefficients tends to increase as the intermediate input ratios increase. In addition, since intermediate input includes the "Self-sector input," representing inter-industrial transactions, which may significantly affect intermediate input ratios, the "Self-sector input" may sometimes be excluded from calculations of "indices of the power of dispersion."

In this case, when only indirect effects excluding the direct effect of 1.0 to the self-sector are considered, they are referred to as the "second category index of the power of dispersion." When effects on the self-sector are completely eliminated and only the effects on the other sectors are considered, they are referred to as the "third category index of the power of dispersion."

(2)   Index of the Sensitivity of Dispersion

The figure for each row in the inverse matrix coefficient table indicates the supplies required directly and indirectly at each row sector when one unit of the final demand for the column sector at the top of the table occurs. The ratio produced by dividing the total (horizontal sum) by the mean value of the entire sum of row will indicate the relative influences of one unit of final demand for a row sector, which can exert the greatest production repercussions on entire industries. This is called the "Index of the Sensitivity of Dispersion," which can be calculated as follows:

$$\text{Index of the sensitivity of dispersion by sector} = \frac{\text{Each sum of row in inverse matrix coefficient table}}{\text{Mean value of the entire horizontal sum in inverse matrix coefficient table}}$$

$$= \frac{b_{i*}}{\overline{B}}$$

Here,

$$b_{i*} = \sum_{j} b_{ij}$$

$$\overline{B} = \frac{1}{n}\sum_{i} b_{i*} = \frac{1}{n}\sum_{i}\sum_{j} b_{ij}$$

(Please refer to Chart 4-5)

The index of the sensitivity of dispersion indicated above is referred to as the "primary index of the sensitivity of dispersion."

Table 4-1 indicates the calculation of index of the sensitivity of dispersion utilizing $[I-(I-\hat{M})A]^{-1}$ as the inverse matrix in the 34-sector table of the 2005 Input-Output Tables. Here, since the sensitivity indices of commerce and transportation, etc. are high, these sectors provide raw materials and services to a wide range of sectors. They are thus sensitive to fluctuations in business cycles in entire industries.

As in the case of the indices of the power of dispersion, the "Self-sector input" may be excluded. Again, as well as the index of the power of dispersion, the "second category index of the sensitivity of dispersion " and "third category index of the sensitivity of dispersion " can be defined.

Since they are based on inverse matrix coefficients, different results may obtain, depending on how sectors are aggregated and on the types of inverse matrix (please refer to Section 7).

Chart 4-5　Inverse Matrix Coefficient Table (Model)

	1	2	3	...	n	Sum of column	Index of the Sensitivity of dispersion
1	b_{11}	b_{12}	b_{13}	⋮	b_{1n}	b_{1*}	$b_{1*}/\overline{B}$
2	b_{21}	b_{22}	b_{23}	⋮	b_{2n}	b_{2*}	$b_{2*}/\overline{B}$
3	b_{31}	b_{32}	b_{33}	⋮	b_{3n}	b_{3*}	$b_{3*}/\overline{B}$
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
N	b_{n1}	b_{n2}	b_{n3}		b_{nn}	b_{n*}	$b_{n*}/\overline{B}$
Sum of row	b_{*1}	b_{*2}	b_{*3}	...	b_{*n}	$\sum b_{i*} = \sum b_{*j}$	
Index of the Power of dispersion	$\dfrac{b_{*1}}{\overline{B}}$	$\dfrac{b_{*2}}{\overline{B}}$	$\dfrac{b_{*3}}{\overline{B}}$	...	$\dfrac{b_{*n}}{\overline{B}}$		

Table 4-1　Tables of Indices of Power of Dispersion and of the Sensitivity of Dispersion for 2005

Sector		Index of Power of Dispersion	Index of the Sensitivity of Dispersion
01	Agriculture, Forestry and Fisheries	0.923177	0.796203
02	Mining	1.007756	0.580723
03	Beverages and Foods	1.043088	0.751185
04	Textile products	1.003350	0.657007
05	Pulp, paper and wooden products	1.102135	1.279113
06	Chemical products	1.150761	1.371140
07	Petroleum and coal products	0.631767	0.992427
08	Ceramic, stone and clay products	0.949884	0.732013
09	Iron and steel	1.375334	1.793105
10	Non-ferrous metals	1.021351	0.955680
11	Metal products	1.104573	0.827734
12	General machinery	1.143948	0.792750
13	Electrical machinery	1.111671	0.672334
14	Information and communication electronics equipment	1.144626	0.540610
15	Electronic parts	1,123348	1.064624
16	Transportation equipment	1.460793	1.097322
17	Precision instruments	1.027784	0.537037
18	Miscellaneous manufacturing products	1.060162	1.357228
19	Construction	1.004194	0.799163
20	Electricity, gas and heat supply	0.847727	1.046403
21	Water supply and waste disposal business	0.857772	0.697185
22	Commerce	0.785642	1.930789
23	Financial and insurance	0.828782	1.792995
24	Real estate	0.648656	0.750270
25	Transport	0.941583	1.723315
26	Information and communications	0.873301	1.393402
27	Public administration	0.756317	0.700739
28	Education and research	0.741392	1.101215
29	Medical service, health and social security and nursing care	0.871414	0.528866
30	Other public services	0.822267	0.560662
31	Business services	0.885475	2.398321
32	Personal services	0.877608	0.562473
33	Office supplies	1.413541	0.565737
34	Activities not elsewhere classified	1.458818	0.650232

(Note) Derived from the 34-Sector Table

(3) Functional Analysis based on Indices of the Power and Sensitivity of Dispersion

By combining the indices of the power of dispersion and those of the sensitivity of dispersion, we can create a typological presentation of the functions of each industrial sector.

As indicated in Chart 4-6, the figures of the sectors are plotted on a chart, with the indices of the power of dispersion on the horizontal axis, and those of the sensitivity of dispersion on the vertical axis. Each position on the chart can reveal characteristics of the industrial sector.

Sectors plotted in Quadrant "I" can both exert strong influence on entire industries and are most affected to external influences. Typically, these are the raw materials manufacturing sectors, including basic materials such as iron and steel, pulp, paper and wooden products, and chemical products.

Quadrant "II" includes sectors whose influence on entire industries is weak, but whose sensitivity is high. Typically, these sectors provide services to other sectors, such as business services, commerce, transportation, and finance and insurance, etc.

Quadrant "III" includes sectors whose influence and sensitivity are both weak; typically, these are primary industrial sectors such as agriculture, forestry, and fisheries, as well as ceramics, stone and clay products, and independent industrial sectors such as real estate, water supply and waste disposal services, etc.

Quadrant "IV" includes sectors with strong influence on entire industries but relatively weak production repercussions. Typically, these sectors involve the manufacture of final goods, including general machinery, textile products, metal products, precision instruments, and construction, etc.

Chart 4-6　Indices of the Power of Dispersion and Sensitivity of Dispersion
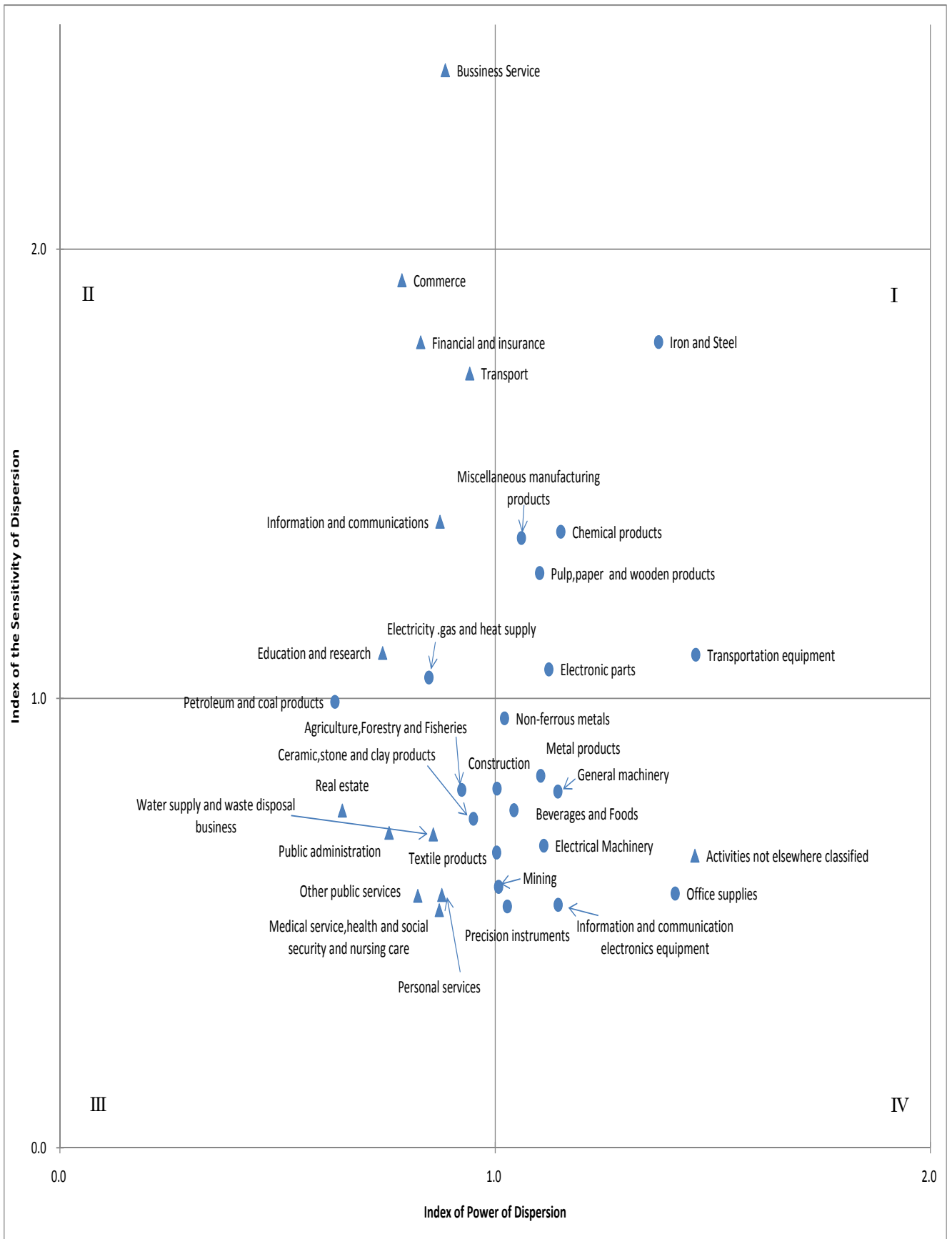
(Note) "●" indicates goods sector,"▲" indicates services sector

§3 Relationship Between Final Demand and Domestic Production

1 Domestic Production Induced by Individual Final Demand Items

Every industry in the endogenous sector supplies goods and services to each industrial sector as well as final demand sectors. On the whole, however, the industrial activities of the endogenous sectors produce to just satisfy the final demand, and their production levels depend on the size of the respective final demands. Based on the competitive import model and when imports fluctuate in proportion to domestic demand, the following relationship holds in the Input-Output Tables, as indicated by equation [10] of §2, through the inverse matrix coefficients:

$$X \qquad = [I-(I-\hat{M})A]^{-1} \quad [I-(I-\hat{M})Y + E]$$

Total domestic products   Inverse matrix   Value of final demand

Here, final demand (F) can be classified be into six categories: [1] consumption expenditure outside households; [2] private consumption expenditures; [3] consumption expenditure of general government; [4] gross domestic fixed capital formation; [5] increase in stock; and [6] exports (E). Domestic products induced by individual final demand items refer to the production of every industry induced by individual final demand items.

Domestic products induced by individual final demand items can bean indication for analyzing and analyzing the items in the final demand that influence value fluctuations in domestic production, and can be calculated as follows:

As mentioned above, the final demand vector F may be divided into domestic final demand vector Y and export vector E. Domestic final demand vector Y can be dissolved into various vectors of domestic final demand items (e.g., private consumption expenditure and gross domestic fixed capital formation, etc.), which may be represented as follows:

$$Y = Y_1 + Y_2 + Y_3 + \ldots Y_n$$

Given that X_K represents the induced production value derived from the respective domestic final demands, domestic final demand may be expressed as follows:

$$X_K = [I-(I-\hat{M})A]^{-1}(I-\hat{M})Y_K$$

$$K = 1,2, \ldots, N$$

Production value induced by exports E can be expressed as follows:

$$X_E = [I-(I-\hat{M})A]^{-1} \quad E$$

Since the aggregate of induced production values by the respective final demand items is equivalent to the value of domestic production, we derive the following equation:

$$X = \sum_{K=1}^{N} X_K + X_E$$

It is also possible to use $(I - A^d)^{-1}$ as the inverse matrix. In that case, the final demand vector multiplying on the right side represents the final demand for domestic items (F^d).

2 Domestic production Inducement Coefficients by Individual Final Demand Items

"Production inducement coefficient by final demand item" is defined as the domestic products induced by individual final demand items divided by the total for corresponding final demand.

Given that:

$$Y_K = \begin{bmatrix} Y_{1K} \\ \vdots \\ Y_{nK} \end{bmatrix}, \qquad X_K = \begin{bmatrix} X_{1K} \\ \vdots \\ X_{nK} \end{bmatrix} \qquad \begin{matrix} K = 1,2,\ldots,N \\ \text{(Domestic final demand items)} \end{matrix}$$

And:

$$E = \begin{bmatrix} E_1 \\ \vdots \\ E_n \end{bmatrix}, \qquad X_E = \begin{bmatrix} X_{1,N+1} \\ \vdots \\ X_{n,N+1} \end{bmatrix}$$

Then, the domestic production of industry "i" induced by domestic final demand item "K" and exports will be X_{ik} and $X_{i,N+1}$, respectively, and the production inducement coefficients can be expressed as follows:

$$\text{Production inducement coefficients by final demand items} = \begin{cases} \dfrac{X_{ik}}{\sum\limits_{j=1}^{n} Y_{jk}} & \text{(Domestic final demand)} \\[20pt] \dfrac{X_{i,N+1}}{\sum\limits_{j=1}^{n} E_j} & \text{(Exports)} \end{cases}$$

This indicates the rate of increase of domestic production in an industry, derived from the total increase of one unit of a certain final demand item (within the same item).

The aggregated production inducement coefficients by final demand items for the respective sectors—that is,

$$\frac{\sum\limits_{j=1}^{n} X_{iK}}{\sum\limits_{j=1}^{n} Y_{jK}} \quad \text{and} \quad \frac{\sum\limits_{i=1}^{n} X_{i,N+1}}{\sum\limits_{j=1}^{n} E_j}$$

—is sometimes known as the production inducement coefficient.

Final demands with higher production inducement coefficients will have greater production repercussion effects. For 2005, exports account for the highest figure.

		Final demand item
		1 2 3 ... ... N, N+1
Industrial sector	1	Production inducement coefficient by final demand item
	2	
	3	
	:	$\left[\dfrac{X_{ik}}{\sum\limits_{j=1}^{n} Y_{jk}} \right]\left[\dfrac{X_{i,N+1}}{\sum\limits_{j=1}^{n} E_j} \right]$
	:	
	n	
Total		

(Note) X_{ik} and $X_{i,N+1}$:   Production inducements by final demand item

$\sum\limits_{j=1}^{n} Y_{jk}, \sum\limits_{j=1}^{n} E_j :$ Total of Final demands

3   Domestic production Inducement Distribution Ratios by Individual Final Demand Items

"Production inducement distribution ratios by final demand items" are defined as the proportion ratios of induced production value derived from the respective industrial sectors. They indicate the degree of influence or weighting of the respective final demand items on the domestic productions in industrial sectors.

		Final demand item	Total
		1 2 3 ... ... N, N+1	
Industrial sector	1	Production inducement distribution ratio by final demand item	
	2		
	3		
	:	$\left[\dfrac{X_{ik}}{\sum\limits_{j=1}^{n} Y_{jk}} \right]\left[\dfrac{X_{i,N+1}}{\sum\limits_{j=1}^{n} E_j} \right]$	1.0
	:		
	n		

(Note) X_{ik} and $X_{i,N+1}$:   Production inducement by final demand item
X_i: Total of production inducement (total domestic products)

§4　Relationship Between Final Demand and Gross Value Added

The domestic production of each sector is comprised of intermediate input and gross value added. Since domestic production can be induced by final demand, we can assume that gross value added, which is part of domestic production, can be similarly induced by final demand.

It is thus possible to apply the relational expression between domestic production and final demand, introduced in §3, to gross value added and final demand in exactly the same manner.

The ratio of gross value added is defined as the gross value added of each sector divided by the domestic production of the sector. This is the gross value added per unit of production, the elements of which can be represented in a diagonal matrix "$\hat{v}$."

$$\hat{v} = \begin{bmatrix} v_1 & & & & 0 \\ & v_2 & & & \\ & & v_3 & & \\ & & & \ddots & \\ 0 & & & & v_n \end{bmatrix} \qquad v_i = \frac{V_i}{X_i}(i=1,2,\cdots,n)$$

Therefore, when V is defined as a vector comprised of gross value added,

$$V = \hat{v}X$$

Thus, the supply-demand balance equation mentioned in §3 can be indicated for the gross value added, as follows:

$$V = \hat{v}\,[I-(I-\hat{M})A]^{-1}[(I-\hat{M})Y+E]$$

This equation can be used to define the following, as in the case of production inducement:

　　[1]　Gross value added inducement

　　[2]　Gross value added inducement coefficient

　　[3]　Gross value added inducement distribution ratio

A characteristic finding from comparisons between the production inducement coefficient and the gross value added inducement coefficient is that "exports" and "gross domestic fixed capital formation," which indicate larger figures among final demand items for production inducement coefficients, give smaller figures than "consumption" for gross value added inducement coefficient. This implies that increasing public sector investment and exports stimulates the economy, but that stimulating consumption is more effective for added value levels (GDP levels).

§5　Relationship Between Final Demand and Imports

1　Imports Induced, Imports Inducement Coefficients and Imports Inducement Distribution Ratios by Individual Final Demand Items

When certain final demands are generated, not all are usually satisfied by domestic production. Some are met by imports.

A fundamental field within input-output analyses is a measurement of the scale of production induced at each sector by generation of a certain final demand. Also critical is determining the scale of imports induced by the same cause. This requires the import coefficient of each sector. The scale of imports induced by one unit of final demand can be calculated with the import coefficients.

In the inverse matrix coefficients based on the $[I-(I-\hat{M})A]^{-1}$ type, commonly utilized in Japan, as explained in §2, the Input-Output Tables do not cover re-exports of imported goods (that is, exports exclude all imports). Thus, import coefficients are defined as ratios to domestic demand, as follows:

$$m_i = \frac{M_i}{\sum\limits_{j=1}^{n} a_{ij}X_j + Y_i} \qquad \hat{M} = \begin{bmatrix} m_1 & & 0 \\ & \ddots & \\ 0 & & m_n \end{bmatrix}$$

$$\therefore M = \hat{M}(AX+Y) \qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots[1]$$

Total domestic products X can be expressed as follows:

$$X = [I - (I - \hat{M})A]^{-1}[(I - \hat{M})Y + E] \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{[2]}$$

The inverse matrix coefficient $[I - (I - \hat{M})A]^{-1}$ is expressed as B and replaces [1] above and expanded as follows:

$$M = \hat{M}AB(I - \hat{M})Y + \hat{M}ABE + \hat{M}Y$$

$$M = [\hat{M}AB(I - \hat{M}) + \hat{M}] \ Y + \hat{M}ABE \quad \cdots\cdots\cdots\cdots\cdots\cdots \text{[3]}$$

In other words, imports can be divided into those induced by domestic final demand, excluding exports (the first term of the right side of M in the equation [3]), and those induced by exports E (the second term on the right side of [3]).

$\hat{M}AB$ can be regarded as the inverse matrix coefficient B multiplied by the input coefficient $\hat{M}A$.

The breakdown of the import inducement by each of the final demand items is presented as the "import inducement coefficient by final demand item." As indicated in equation (3) of the above §1, imports M can be resolved as follows:

$$M = [\hat{M}AB(I - \hat{M}) + \hat{M}] \ Y + \hat{M}ABE$$

As is apparent from this equation, these factors are given by multiplying the final demands of the relevant items, respectively. Namely, they are given by multiplying the respective final demand item vectors from the "consumption expenditure outside households" to "increase in stocks," which are domestic final demands by the matrix $[\hat{M}AB(I - \hat{M}) + \hat{M}]$, and for "exports" by multiplying the export vector by the matrix $\hat{M}AB$.

Import inducement coefficients by final demand items and import inducement distribution ratios by final demand items are not explained here, as they can be calculated in the same as in the case of production inducement coefficients and production inducement distribution ratios in §3.

2   Comprehensive Imports Coefficients

The sum of column of the matrix $[\hat{M}AB(I - \hat{M}) + \hat{M}], \hat{M}AB$ are coefficients that indicate the size of import inducements due to generation of one unit of "final demand excluding exports" and "exports" (the same itemized structure), and are referred to as "comprehensive import coefficients." The figures are taken from 190 sectors and 108 sectors in the Data Report (2).

§6   Labor Input-Output Analysis Coefficients

1   Labor Inducement Coefficients

In the Input-Output Tables, the following relationship holds between domestic production and final demand with, suggesting inverse matrix coefficients:

$$X = [I - (I - \hat{M})A]^{-1}[(I - \hat{M})Y + E] \quad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\text{[1]}$$

X: Total domestic products

$[I - (I - \hat{M})A]^{-1}$: Inverse matrix

$[(I - \hat{M})Y + E]$: Final demand

Here, each row of matrix L of the labor input (man-year) for each sector is divided by domestic production to give the labor input coefficient matrix L'.

(Labor input L)

	Sector 1	Sector 2	Sector 3	.....	Sector n	
Total employees	l_{11}	l_{12}	l_{13}	.....	l_{1n}	
Self-employed	l_{21}	l_{22}	l_{23}	.....	l_{2n}	Table on Employees Engaged in
Family worker	l_{31}	l_{32}	l_{33}	.....	l_{3n}	Production Activities
:	:	:	:	.....	:	(by Occupation)
:	:	:	:	.....	:	
:	:	:	:	.....	:	
Total domestic products	X_1	X_2	X_3	.....	X_n	

(Labor input coefficient, L')

	Sector 1	Sector 2	Sector 3	.....	Sector n	
Total employees	l'_{11}	l'_{12}	l'_{13}	.....	l'_{1n}	Table on Employees Engaged in
Self-employed	l'_{21}	l'_{22}	l'_{23}	.....	l'_{2n}	Production Activities
Family worker	l'_{31}	l_{32}	l'_{33}	.....	l'_{3n}	(by Occupation)
:	:	:	:	.....	:	
:	:	:	:	.....	:	
:	:	:	:	.....	:	

(Note) $l'_{ij} = \dfrac{l_{ij}}{X_j}$

Here, the total number of employees and the i-th employee position are analyzed. The i-th row of L is placed vertically to produce vector L_i, and the i-th element of L' is placed diagonally to produce matrix $\hat{L}'_i$, as follows:

$$L_i = \begin{bmatrix} l_{i1} \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ l_{in} \end{bmatrix}, \quad \hat{L}'_i = \begin{bmatrix} l'_{i1} & & & & & 0 \\ & l'_{i2} & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ 0 & & & & & l'_{in} \end{bmatrix}$$

$L_i = \hat{L}'_i X$

$\quad = \hat{L}'_i [I - (I - \hat{M})A]^{-1} [(I - \hat{M})Y + E]$

$\quad = \hat{L}'_i B \, [(I - \hat{M})Y + E]$ .... [2]

Here, the following equation is defined.

$\quad B = [I - (I - \hat{M})A]^{-1}$

Each column of the matrix $\hat{L}'_i B$ indicates the size of labor demand required directly and indirectly at each sector when one unit of final demand is generated for each sector. The elements of this matrix $\hat{L}'_i B$ are commonly referred to as "labor inducement coefficients."

Each row of matrix $L'B$ indicates the scale of labor demand by occupational positions required directly and indirectly when one unit of final demand is generated for each sector. This may also be referred to as "labor inducement coefficients." "Occupation inducement coefficients" to be explained later are based on the latter concept.

Domestic final demand Y is comprised of consumption expenditure of households, consumption expenditure of general government, gross domestic fixed capital formation and exports, etc., and can be expressed as follows:

$\quad Y = Y_1 + Y_2 + \ldots + Y_m$.. [3]

From [2] and [3], the following equation can be obtained:

$\quad L_i = \hat{L}_i B \, [(I - \hat{M})(Y_1 + \cdots + Y_m) + E]$

$\quad\quad = \hat{L}'_i B (I - \hat{M}) Y_1 + \cdots + \hat{L}'_i B (I - \hat{M}) Y_m$

$\quad\quad\quad\quad + \hat{L}'_i B E$ .... [4]

Each term on the right-hand side indicates the comprising item of the final demand of labor induced.

In input-output analyses, it is assumed that input coefficients are stable and that no significant differences exist among them between the time at which the tables are compiled and the time at which analyses are made. A similar assumption is applied to labor-related input-output analyses; labor input coefficients are assumed to be stable.

However, unlike input coefficients, labor input coefficients cannot always be stable. For instance, even if production in a certain sector has doubled, the labor input does not necessarily double when industrial robots are installed or the operating ratios are improved. In conducting labor-related input-output analyses, therefore, it is necessary to fully consider changes in operating ratios and labor productivity.

2 Labor-Related Indices of Power and Sensitivity of Dispersion

As the indices of the power of dispersion and those of the sensitivity of dispersion can be obtained from the inverse matrix coefficients, the indices of the power of dispersion and those of the sensitivity of dispersion concerning labor inducements can also be obtained from the labor inducement coefficient matrix $\hat{L}_i'B$.

(1) Index of the power of dispersion for labor inducement

This index is used to compare the sizes of effects at different sectors of an increase of one unit of final demand at a certain sector on labor demand at the respective row sectors.

The "primary index of the power of dispersion for labor inducement" can be calculated as follows:

Primary index of the power of dispersion for labor inducement by sector

$$= \frac{\text{Each vertical sum of labor inducement coefficient matrix}}{\text{Mean of the entire vertical sum of labor inducement coefficient matrix}}$$

$$= \frac{C_j}{\overline{C}}$$

Here, $C = \hat{L}_i'B = [C_{ij}]$

$$C_j = \sum_i C_{ij}, \quad \overline{C} = \frac{1}{n}\sum_j C_j$$

The bigger the index of the power of dispersion, the greater the labor demand at each sector, induced by one unit of final demand at the sector.

While the "primary index of the power of dispersion for labor inducement" indicates the direct and indirect effects of labor inducement, including the self-sector, the "tertiary index of the power of dispersion for labor inducement" completely eliminates the effects on the self-sector and concentrates on labor inducement effects on the other sectors. It is calculated by replacing the diagonal element on the labor inducement coefficient matrix with zero, and using a similar method to that applied for the primary index of the power of dispersion. The bigger the index of the tertiary index of the power of dispersion, the greater the labor inducement effects on the other sectors.

(2) Index of the sensitivity of dispersion for labor inducement

The index of the power of dispersion is calculated from each vertical sum of the labor inducement coefficients. An index can also be calculated from each horizontal sum of the labor inducement coefficients, which is referred to as the "index of the sensitivity of dispersion." Used to compare labor inducement effects received at different sectors from one unit of final demand generated at each sector, the "primary index of the sensitivity of dispersion for labor inducement" is calculated as follows:

Primary index of the sensitivity of dispersion for labor inducement by sector

$$= \frac{\text{Each horizontal sum of labor inducement coefficient matrix}}{\text{Mean of the entire horizontal sums of labor inducement coefficient matrix}}$$

$$= \frac{C_i}{\overline{C}}$$

Here, $C_i = \sum_j C_{ij}, \quad \overline{C} = \frac{1}{n}\sum_i C_i$

Sectors indicating higher "primary indices of the sensitivity of dispersion for labor inducement" are more susceptible to labor inducement effects.

The "tertiary index of the sensitivity of dispersion for labor inducement" indicates the relative effects of labor inducement on each sector, excluding the self-sector, due to generation of one unit of final demand.

3 Occupation Inducement Coefficients

The Employment matrix (table on employees engaged in production activities [by occupation]) makes it possible to calculate the employment inducement coefficient by occupation.

The occupational input coefficient matrix can be derived by dividing each element of the employment matrix S by domestic production at each sector.

(Employment Matrix S)

	Sector 1	Sector 2	Sector 3	.....	Sector n
Occupation 1	S_{11}	S_{12}	S_{13}	.....	S_{1n}
Occupation 2	S_{21}	S_{22}	S_{23}	.....	S_{2n}
Occupation 3	S_{31}	S_{32}	S_{33}	.....	S_{3n}
:	:	:	:	.....	:
:	:	:	:	.....	:
:	:	:	:	.....	:
Domestic production	X_1	X_2	X_3	.....	X_n

Table on Employees Engaged in Production Activities (by Occupation)

(Note) Employees include paid officers.

(Occupation Input Coefficient *S'*)

	Sector 1	Sector 2	Sector 3	.....	Sector n
Occupation 1	S'_{11}	S'_{12}	S'_{13}	.....	S'_{1n}
Occupation 2	S'_{21}	S'_{22}	S'_{23}	.....	S'_{2n}
Occupation 3	S'_{31}	S'_{32}	S'_{33}	.....	S'_{3n}
:	:	:	:	.....	:
:	:	:	:	.....	:
:	:	:	:	.....	:

(Note) $S'_{ij} = \dfrac{S_{ij}}{X_j}$

The vector S^* comprised of the sum of row of S may be expressed as follows:

$$S^* = S'B \left[(I - \hat{M})Y + E\right] \text{...} [5]$$

Here, $B = [I - (I - \hat{M})A]^{-1}$

The matrix $S'B$ is the "occupation inducement coefficients" matrix, representing the number of employees by occupation, to be required directly and indirectly by one unit of final demand at each sector.

4   Labor and Occupation Induced Coefficients by Individual Final Demand Items

As stated earlier, domestic final demand Y can be resolved for each item and represented as follows:

$$Y = Y_1 + Y_2 + \cdots \cdots + Y_m \text{.......................................} [3]$$

$$L_i = \hat{L}'_i B(I - \hat{M})Y_1 + \cdots + \hat{L}'_i B(I - \hat{M})Y_m$$
$$+ \hat{L}'_i BE \text{................................} [4]$$

The above equations can be used to obtain the labor inducement coefficient by final demand items. They can also indicate which final demand items and how many employees or workers in the respective sectors will be required, as well as their respective occupational positions.

In the equation [5], final demands can be resolved for the respective items, as follows:

$$S^* = S'B(I - \hat{M})Y_1 + \cdots + S'B(I - \hat{M})Y_m + S'BE$$

This obtains the number of employees by occupations required for specific final demand items (occupation inducement coefficients by final demand items).

§7   Problem of Sector Integration

1   Introduction

In the 2000 Input-Output Tables, the 188-sector tables, the 104-sector tables, 32-sector tables and 13-sector classification tables were compiled based on a basic sector classification comprised of 517 rows and 405 columns. In addition, users can easily compile aggregated sector classification tables for their own purposes just by adding up the figures of relevant sectors:

If the objective is to read the Input-Output Tables as they are, sector integration is simply how accuracy the tabulations should be. However, the most important thins in using Input-Output Tables is conducting economic forecasts, measuring the mode of specific economic policies, or analyzing prices using input coefficients, inverse matrix coefficients, production inducement coefficients by final demand item, etc. If Input-Output Tables are to be useful for these purposes, the manner in which the sectors for Input-Output Tables are defined will be crucial.

That is, for calculations of production inducement and other effects with Input-Output Tables (to calculate inverse matrix coefficients), different results are generally obtained from different sector establishments.

This was once pointed out by W. Leontief, founder of Input-Output Tables, as follows:

Industrial classifications for input-output analyses are led by considerations of technical homogeneity.Integration problems may arise from scaling down the matrix by integrating the columns in input-output matrix and the related several rows. The relationship between the nature of the integrated matrix and that of the non-integrated ones depends on the positions at which the input columns of the integrated sectors are placed within the non-integrated matrix. Under certain ideal conditions, the integrated inverse matrix of the original matrix corresponds to the inverse matrix of the integrated matrix. If these conditions are met not completely but approximately, that correspondence has been realized only approximately.

Which sector should be established to eliminate production repercussions? What needs to be kept in mind when integrating sectors? These points will be addressed in the following sections.

2　Theoretical Aspects of Sector Integration

(1)　Integration of two sectors

We will discuss a case of integrating Sector 1 and Sector 2 by defining an input coefficient matrix A, as follows:

$$
A = \begin{bmatrix}
P & \vdots & u_1 & u_2 & \vdots & R \\
\cdots & \cdots & \cdots & \cdots & \vdots & \cdots \\
l_1' & \vdots & a_{11} & a_{12} & \vdots & r_1' \\
l_2' & \vdots & a_{21} & a_{22} & \vdots & r_2' \\
\cdots & \vdots & \cdots & \cdots & \cdots & \cdots \\
Q & \vdots & d_1 & d_2 & \vdots & S
\end{bmatrix}
\begin{matrix}
\text{Sector } l \\ \\
\text{Sector } 1 \\
\text{Sector } 2 \\ \\
\text{Sector } r
\end{matrix}
$$

（with column headers: Sector l　Sector 1　Sector 2　Sector r）

Here, X_1 and X_2 are defined as domestic productions of Sector 1 and Sector 2, respectively, and the following relationships are established.

$$
\alpha = \frac{X_1}{X_1 + X_2} \qquad \beta = \frac{X_1}{X_1 + X_2}
$$

In this case, the input coefficient matrix when Sector 1 and Sector 2 are integrated can be represented in the following matrix:

$$
\overset{+}{A} = \begin{bmatrix}
P & \vdots & au_1 + \beta u_2 & \vdots & R \\
\cdots & \cdots & \cdots & \vdots & \cdots \\
l_1' + l_2' & \vdots & \begin{matrix} \alpha(a_{11} + a_{21}) \\ + \beta(a_{12} + a_{22}) \end{matrix} & \vdots & r_1' + r_2' \\
\cdots & \vdots & \cdots & \cdots & \cdots \\
Q & \vdots & \alpha d_1 + \beta d_2 & \vdots & S
\end{bmatrix}
$$

Here, final demand can be expressed as follows:

$$
F = \begin{bmatrix} F_l \\ F_2 \\ F_3 \\ F_r \end{bmatrix}
$$

F_l: Final demand for Sector l
F_1: Final demand for Sector 1
F_2: Final demand for Sector 2
F_r: Final demand for Sector r

In the above inverse matrix model considering, the conditions required to make production inducements in A and $\overset{+}{A}$ identical for a certain final demand F.

First, the input coefficient matrix A prior to the sector integration is used to calculate the primary repercussion on final demand F. When X^1 is defined as the vector of domestic production induced by the primary repercussion on the relevant sectors, the following can be defined:

$$X^1 = \begin{bmatrix} X_l^1 \\ X_1^1 \\ X_2^1 \\ X_r^1 \end{bmatrix} = AF = \begin{bmatrix} PF_l + u_1 F_1 + u_2 F_2 + RF_r \\ l_1' F_l + a_{11} F_1 + a_{12} F_2 + r_1' F_r \\ l_2' F_l + a_{21} F_1 + a_{22} F_2 + r_2' F_r \\ QF_l + d_1 F_1 + d_2 F_2 + SF_r \end{bmatrix} \quad \text{...............[1]}$$

Next, the input coefficient matrix A after the sector integration is used to calculate the primary repercussion on final demand F:

Here, $\overset{+}{F} = \begin{bmatrix} F_l \\ F_1 + F_2 \\ F_r \end{bmatrix}$.

When X_1 is defined as the vector of domestic production induced by the primary repercussion on the relevant sectors, we can define the following:

$$X^1 = \begin{bmatrix} \overset{+}{X}_l{}^1 \\ \overset{+}{X}{}^1_{1+2} \\ \overset{+}{X}_r{}^1 \end{bmatrix} = \overset{+}{A}\,\overset{+}{F} = \begin{bmatrix} PF_l + \\ (l_1' + l_2')F_l + \\ QF_l + \end{bmatrix}$$

$$\begin{bmatrix} (\alpha u_1 + \beta u_2)(F_1 + F_2) & + RF_r \\ \{\alpha(a_{11} + a_{21}) + \beta(a_{12} + a_{22})\}(F_1 + F_2) + (r_1' + r_2')F_r \\ (\alpha d_1 + \beta d_2)(F_1 + F_2) & + SF_r \end{bmatrix} \quad \text{...........[2]}$$

Here, regardless of the status of integration, any F should meet the following conditions to make production inducements by the primary repercussion coincide:

$$\begin{aligned} X_l^1 &= \overset{+}{X}_l{}^1 \\ X_1^1 + X_2^1 &= \overset{+}{X}{}^1_{1+2} \\ X_r^1 &= \overset{+}{X}_r{}^1 \end{aligned} \qquad \text{...[3]}$$

If we substitute [1] and [2] into [3], we obtain the following from $\alpha + \beta = 1$.

$$\begin{aligned} u_1 &= u_2 \\ a_{11} + a_{21} &= a_{12} + a_{22} \\ d_1 &= d_2 \end{aligned} \qquad \text{...[3]'}$$

As mentioned above, the equations in [3]' indicate conditions under which sector integration does not affect the magnitude of the primary repercussions. They can also be the conditions for the coincidence of the domestic production inducements, X^2 and $\overset{+}{X}{}^2$, due to the secondary repercussions obtained by replacing F of [1] and $\overset{+}{F}$ of [2] into X^1 and $\overset{+}{X}{}^1$, respectively, and furthermore the conditions for the coincidence of the sizes of the ultimate repercussions (so-called "production inducements"). The conditions under which integration will not change production inducements at each sector specified in [3]'; that is, input coefficients of the respective sectors to be integrated should coincide with the input coefficients of the relevant sectors after integration. In other words, there are no changes in production inducements before and after integration only when the input coefficients representing the technological structures for production are identical.

Classifications of sectors in the Input-Output Tables for Japan are based on activities relating the types of goods and services. The above conditions indicate that the activity-based homogeneity is required for defining sectors. In this sense, they indicate the criteria and principles of section definition.

(2)　Effects of production inducements on other sectors due to sector integration

Next, effects of sector integration on production inducements of other sectors will be considered. Here, to simplify the discussion, a certain sector (sector " l ") is represented all sectors.

The conditions under which the sizes of primary repercussions before and after sector integration are identical are the ones give below from (3) above.

$$X_l^1 = \overset{+}{X_l^1}$$

The condition derived from the above is:

$$u_1 = u_2$$

In other words, when the production coefficients from Sector l to Sector 1 and Sector 2 to be integrated are identical, the primary production repercussions on Sector l due to any final demand are identical before and after sector integration. However, for second and further repercussions, they generally do not coincide before and after integration.

Specifically, when the following can be defined,

$$u_1 = u_2 = 0 \quad \text{and} \quad R = 0$$

or, when sectors other than Sector l, which is under study, do not receive any input from Sector l, while sectors other than Sector l are integrated, no effects will be found in production inducements to Sector l.

A clearer overall picture of these relationships can be provided by blocking the input coefficient table modified as follows by maintaining the relationships between, and at the same time changing the orders of, the row and column sectors of the input coefficient tables.

	I	II	III	IV
I	× × ×			
II		× × × × × × × × × ×		
III			× × × ×	
IV	× × × × × × × × ×	× × × × × × × × × × × ×	× × × × × ×	× × × × × × × × ×

(Note) All except "×" are "0."

Here, to analyze the repercussion effects from a certain final demand, for instance, only concerning Group I, regardless how Groups II, III, and IV are integrated, the inducement effects at I are held constant. The same is true of Group II or Group III.

Or, when the relative ratios of final demands at the sectors to be integrated are equivalent to the respective domestic production ratios—that is, the following relations can be established:

$$F_1:F_2 = X_1:X_2 = \alpha : \beta \quad (\alpha + \beta = 1)$$

Here, the following can be defined.

$$X^1 = \begin{bmatrix} PF_l + (u_1 + \dfrac{\beta}{\alpha}u_2)F_1 + RF_r \\[2mm] l_1'F_l + (a_{11} + \dfrac{\beta}{\alpha}u_{12})F_1 + r_1'F_r \\[2mm] l_2'F_1 + (a_{21} + \dfrac{\beta}{\alpha}a_{22})F_1 + r_2'F_r \\[2mm] QF_l + (d_1 + \dfrac{\beta}{\alpha}d_2)F_1 + SF_r \end{bmatrix}$$

$$\overset{+}{X^1} = \begin{bmatrix} PF_l \quad + (\alpha u_1 + \beta u_2) \\[2mm] (l_1' + l_2')F_l + \{\alpha(a_{11} + a_{21}) + \beta(a_{12} + a_{22})\} \\[2mm] QF_l \quad + (\alpha d_1 + \beta d_2) \end{bmatrix}$$

$$\times \quad (1+\frac{\beta}{\alpha})F_1 + RF_r \; \Big]$$
$$\times \quad (1+\frac{\beta}{\alpha})F_1 + (r_1' + r_2')F_r$$
$$\times \quad (1+\frac{\beta}{\alpha})F_1 + SF_r \; \Big]$$

$$= \begin{bmatrix} PF_1 & + (u_1 + \dfrac{\beta}{\alpha}u_2)F_1 \\ (l_1' + l_2')F_l + \left\{ (a_{11} + a_{21}) + \dfrac{\beta}{\alpha}(a_{12} + a_{22})F_1 \right\} \\ QF_l & + (d_1 + \dfrac{\beta}{\alpha}d_2)F_1 \end{bmatrix}$$

$$+ \quad RF_r \; \Big]$$
$$+ \quad (r_1' + r_2')F_r$$
$$+ \quad SF_r \; \Big]$$

In other words, integrated X^1 corresponds to $\overset{+}{X}{}^1$.

(3) Conditions for preventing production repercussion effects due to integration

The following conclusions summarize the above:

[1]   When the input coefficients of the sectors to be integrated are identical to the input coefficients of the sectors after integration, production repercussions are completely identical for any final demand.

[2]   When the input coefficients of the sectors to be integrated from the other specific sectors do not change before and after sector integration, the primary production repercussions on the specific sectors have not been changed with respect to any final demand.

[3]   For sectors that have not received any input from certain specified sectors, whatever integration may take place, there is no effect on production repercussions on the specified sector.

[4]   When the mutual ratios of the final demands at the sectors to be integrated are equal to those of the respective domestic productions, the primary production repercussions due to the final demands are identical in all relevant sectors.

Furthermore, when considering the inverse matrix model that accounts for imports, except for [3] above, another condition is added: that import ratios of the sectors to be affected by $[I - (I - \hat{M})A]^{-1}$ of the integration are equal. In this manner, except for such highly unusual cases in which input structures do not change before and after integration, it should always be kept in mind that the integration (or establishment) of sectors may cause different results to production repercussions and inducements.

3   Example of Sector Integration

Effects of example sector integration will be investigated using the 2005 Input-Output Table. The following two methods are used to calculate production inducements (by final demand item) of the 13 sectors and compare the results. The $[I - (I - \hat{M})A]^{-1}$ type inverse matrix coefficient is used.

[1]   Calculations are conducted with 190 sectors, then the results are integrated into 13 sectors.

[2]   Calculations are conducted with 13 sectors from the beginning.

The comparison results are as indicated in Table 4-2, the figures represent the difference ratios of [2] against [1]. These figures make it quite clear that significant differences exist, particularly in the agriculture, forestry, and fisheries sector and in the mining sector, suggesting notable effects from sector integration. In addition, looking at the weighted average figures of the absolute values of the above ratios by the weights of production inducements derived from [1] for each row and column (which are referred to as "deviation rates"), consumption expenditure outside households and exports indicate greater figures for the respective final demand items.

Furthermore, instead of [2] above, the following comparisons with [1] have been conducted similarly:

[2]'   After calculations are conducted with the 34 sectors, the results are integrated into 13 sectors.

[2]"   After calculations are conducted with 108 sectors, the results are integrated into 13 sectors.

The results can be presented only in the form of deviation rates by final demand items, as in Table 4-3.

4　Summary

In Section 3 above, the integration to 13 sectors was reviewed for the sake of convenience. In actual analyses, however, integration is commonly conducted to 32 or more sectors. Still, the basic premise remains the same.

Given the recent remarkable progress in computing power, it is now recommended that integration be conducted after calculating as many sectors as possible. Computations should at least be performed for sector tables one stage higher that the one required for analysis at hand, specifically, when the results need to be compared for final demand items and respective sectors. However, in sector integration within the scope in which conditions specified in "2" may be satisfied even approximately, the repercussion effects are not exceptional. Specifically, when only certain sectors are analyzed, "blocking" may realize effective sector integration.

Table 4-2　Difference in Production Inducement due to Sector Integration (Difference Ratio)

(unit：%)

	Consumption expenditure outside households	Consumption expenditure (private)	Consumption expenditure of general government	Gross domestic fixed capital formation	Increase in stocks	Exports total	Deviation rate (λ_{i*})
01　Agriculture, forestry and fishery	−62.78	−34.99	88.63	281.82	−8.60	710.07	63.51
02　Mining	2,519.73	242.25	277.44	−67.29	−101.70	155.89	104.39
03　Manufacturing	−3.40	12.01	11.57	−2.00	−9.17	−10.80	8.82
04　Construction	12.74	−3.30	3.45	0.18	4.64	2.76	0.60
05　Electricity, gas and water supply	−30.57	−2.42	5.12	19.04	−7.62	−1.84	5.35
06　Commerce	−18.43	−0.10	4.14	1.52	2.95	0.89	1.26
07　Finance and insurance	−4.22	2.60	21.42	−14.56	−10.07	−6.15	5.89
08　Real estate	−17.13	0.41	26.09	−9.10	−10.00	−19.03	1.38
09　Transport	−12.19	2.56	4.09	−5.51	−4.71	−1.27	3.18
10　Information and communications	6.60	−2.93	39.32	−0.89	−6.54	−19.08	6.74
11　Public administration	0.22	1.31	0.05	−6.50	−8.90	−8.46	0.18
12　Services	6.56	4.19	−2.25	−10.39	9.57	−8.34	4.66
13　Activities not elsewhere classified	0.22	3.40	15.35	−6.50	−8.91	−8.46	6.17
Deviation rate (λ_{*j})	10.35	5.13	4.63	4.12	9.93	11.13	5.94

Note: i : Industrial sectors,　j : Final demand sectors

　　Z ij is calculated by 190 sectors and is integrated by 13 sectors.

　　Z'ij is calculated by 13 sectors.

　Difference ratio :　$\rho_{ij} = (Z'_{ij}/Z_{ij} - 1) \times 100$

　Deviation rate :　$\lambda_{i*} = \sum_j \left(|\rho_{ij}| \times \frac{Z_{ij}}{\sum_j Z'_{ij}} \right)$　　　　$\lambda_{*j} = \sum_i \left(|\rho_{ij}| \times \frac{Z_{ij}}{\sum_i Z'_{ij}} \right)$

　　　　　　　$\lambda_{ij} = \sum_{ij} |\rho_{ij}| \times \frac{Z_{ij}}{\sum_{ij} Z'_{ij}}$

Table 4-3　Deviation Rates by Final Demand Item at Each Aggregated sector

(unit：%)

	Consumption expenditure outside households	Consumption expenditure (private)	Consumption expenditure of general government	Gross domestic fixed capital formation	Increase in stocks	Exports total	Deviation rate (λ_{i*})
[2]　　(13/190)	10.35	5.13	4.63	4.12	9.93	11.13	5.94
[2]'　　(34/190)	8.96	1.31	2.78	2.86	9.95	2.69	2.34
[2]"　(108/190)	0.63	0.38	0.40	0.93	5.22	1.04	0.64